(1)

Polynomial Representation of Convolutional Codes

$u = \cdots u_{-1}\, u_0\, u_1\, u_2 \cdots$

$c = \,, c_{-1}\, c_0\, c_1\, c_2$

$u(D) = \cdots u_{-1} D^{-1} + u_0 + u_1 D + u_2 D^2 + \cdots$

$c(D) = \cdots + c_{-1} D^{-1} + c_0 + c_1 D + c_2 D^2 + \cdots$

$$G = \begin{pmatrix} G_0 & G_1 & \cdots & G_m & & \\ & G_0 & G_1 & \cdots & G_m & \\ & & G_0 & \cdots & \cdots & G_m \\ & & & \ddots & & \ddots \end{pmatrix}$$

$c = uG$

$c(D) = u(D)\, G(D)$

$G(D) = G_0 + G_1 D + \cdots + G_m D^m$

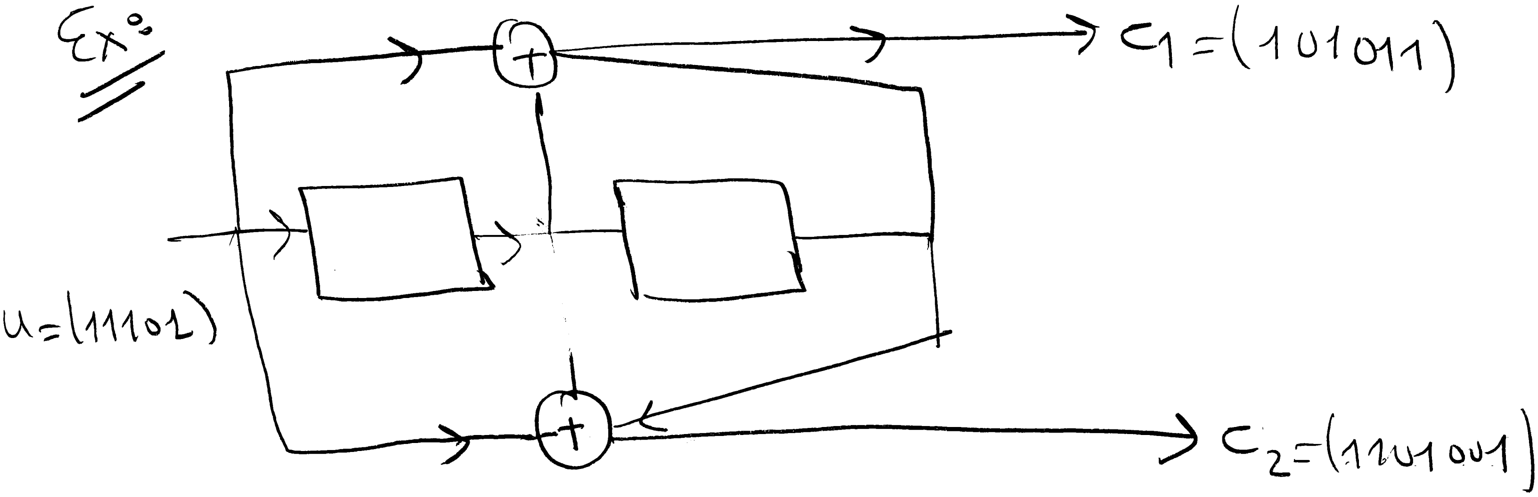
$$c_j(D) = \sum_{i=1}^{k} u_i(D)\, g_{ij}(D)$$

$g_{ij}(D) \to$ generator polynomial from i (input) to output (j)

$G(D) = \left(g_{ij}(D) \right)$

(2) $G(D) = \begin{pmatrix} G_1^1(D) & G_1^2(D) & \cdots \\ G_2^1(D) & G_2^2(D) & \cdots \\ \vdots & & \ddots \end{pmatrix}$

Ex:



$(2,1,2) \rightarrow$ convolutional code (encoder)

$n \quad k \quad m$

Impulse responses?

G_0, G_1? $\quad G = ?$

$G(D) = ?$

Sln:

$h^{11} = (1\ 1\ 1) \qquad h^{12} = (1\ 0\ 1)$

$G_0 = (1\ 1) \quad G_1 = (1\ 0) \quad G_2 = (1\ 1)$

$$G = \begin{pmatrix} 11 & 10 & 11 & & & & & \\ 00 & 11 & 10 & 11 & & & & \\ 00 & 00 & 11 & 10 & 11 & & & \\ 00 & 00 & 00 & 11 & 10 & 11 & & \\ 00 & 00 & 00 & 00 & 11 & 10 & 11 & \end{pmatrix}$$

2A

$$G(D) = G_0 + G_1 D + \cdots G_m D^m$$

$$G(D) = (1\,1) + (1\,0)D + (1\,1)D^2$$

$$= (1 + D + D^2 \quad 1 + D^2)$$

OR impulse ~~responses~~ are

$$h^{11} = (1\,1\,1) \qquad h^{12} = (1\,0\,1)$$

$$h^{11}(D) = 1 + D + D^2 \qquad h^{12}(D) = 1 + D^2$$

$$g^{11}(D) = 1 + D + D^2 \qquad g^{12}(D) = 1 + D^2$$

called generator polynomials

$$G(D) = (g_{ij}(D))$$

$$G(D) = (1 + D + D^2 \quad 1 + D^2)$$

(2B)

$g^{11}(D) = 1 + D + D^2 \qquad g^{12}(D) = 1 + D^2$

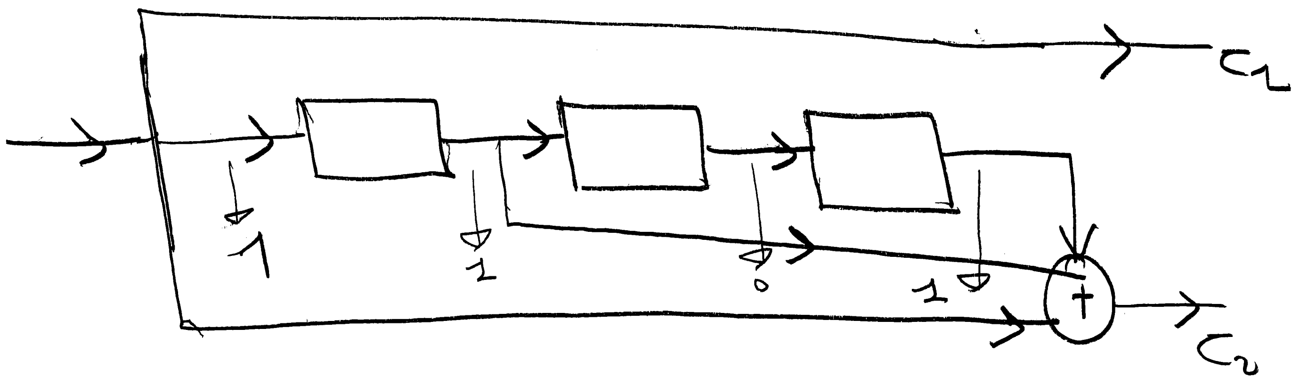
$G(D) = \begin{pmatrix} 1+D+D^2 & 1+D^2 \end{pmatrix}$

$u(D) = \begin{pmatrix} 1+D+D^2+D^4 \end{pmatrix}$

$c(D) = u(D)\, G(D)$

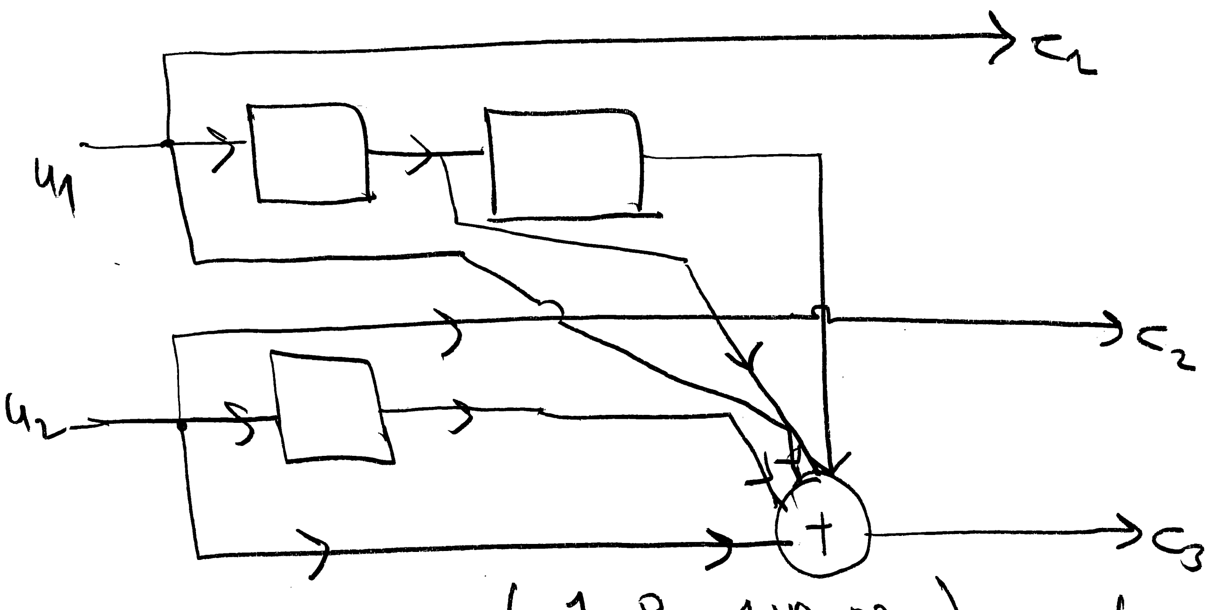
$c_1(D) = u(D)\, g_{11}(D) \qquad c_2(D) = u(D)\, g_{12}(D)$

Ex:



$G(D) = \begin{pmatrix} 1 & 1+D+D^3 \end{pmatrix}$

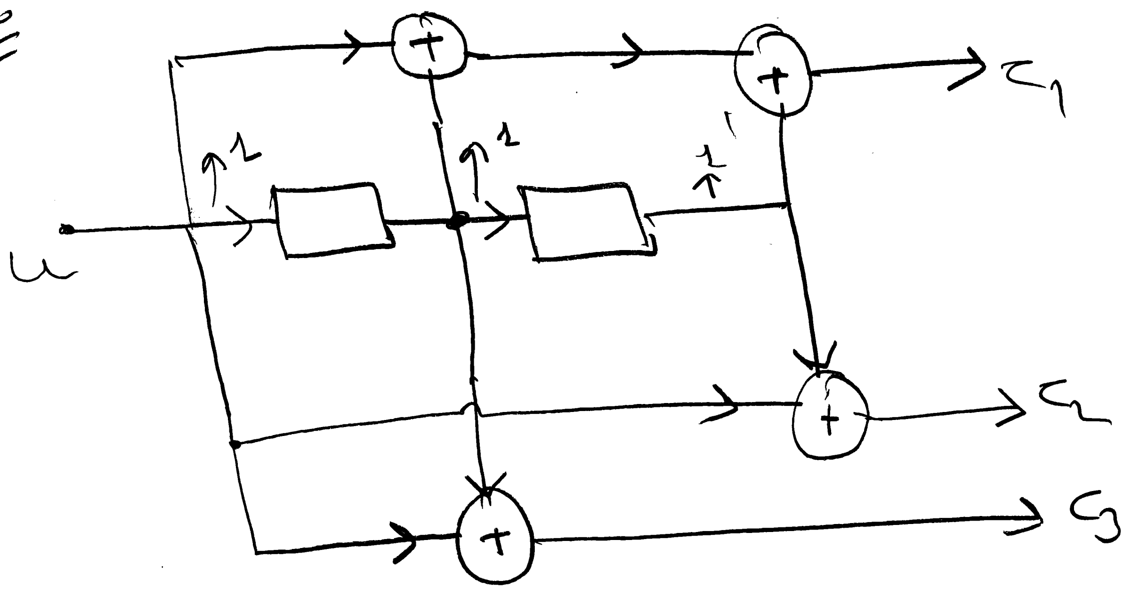
Ex:



$G(D) = \begin{pmatrix} 1 & 0 & 1+D+D^2 \\ 0 & 1 & 1+D \end{pmatrix} \to H(D) = \begin{pmatrix} 1+D+D^2 & 1+D & 1 \end{pmatrix}$

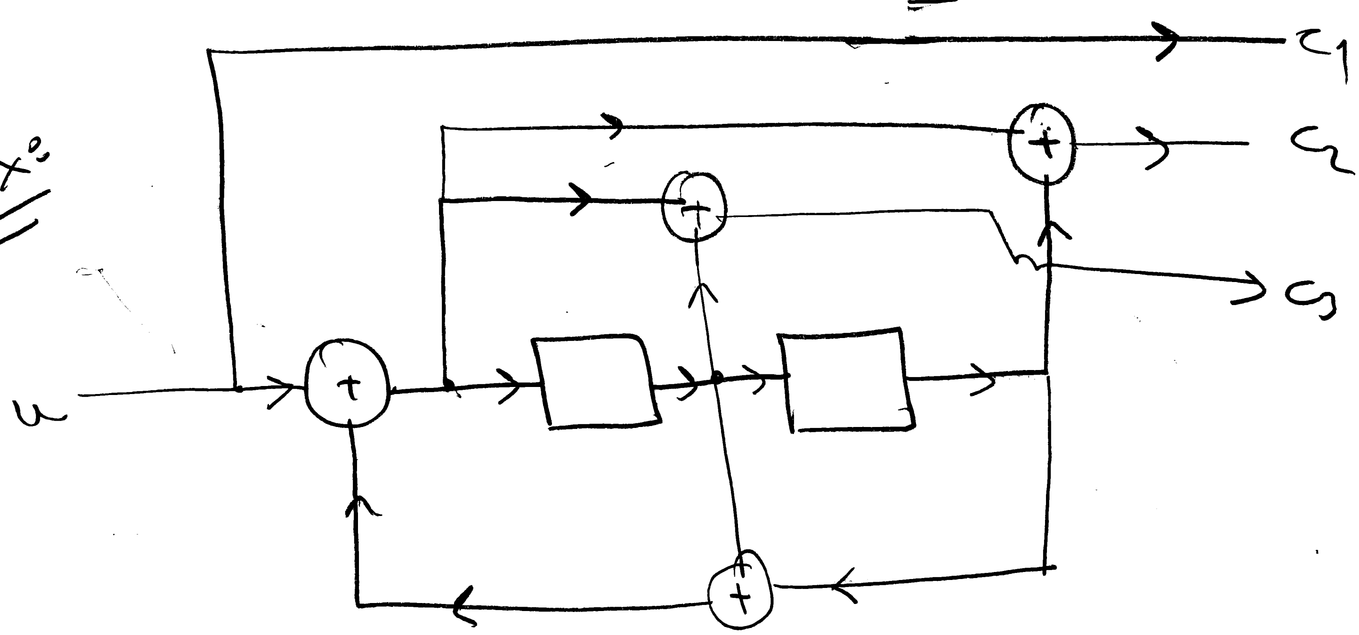
(3)

Ex :



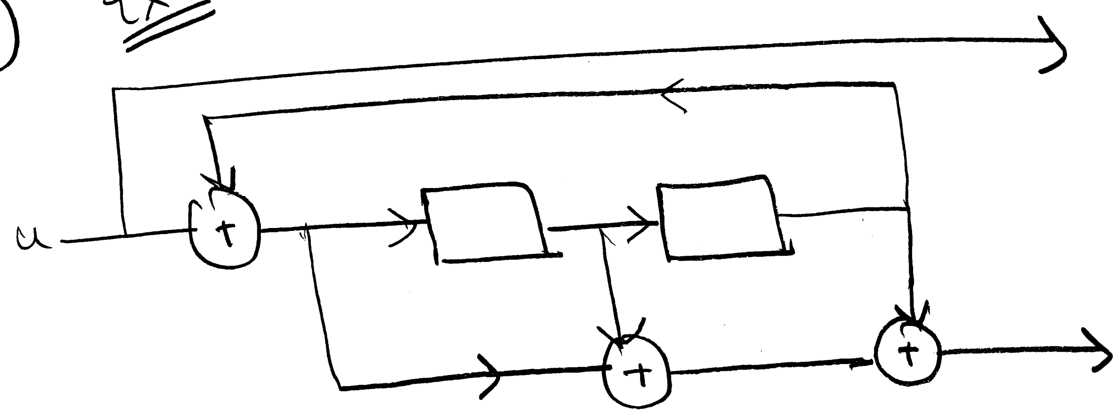
$$G(D) = \left[\, 1+D+D^2 \quad 1+D^2 \quad 1 \,\right]$$

Ex :



$$G(D) = \left[\, 1 \quad \frac{1+D^2}{1+D+D^2} \quad \frac{1+D}{1+D+D^2} \,\right]$$

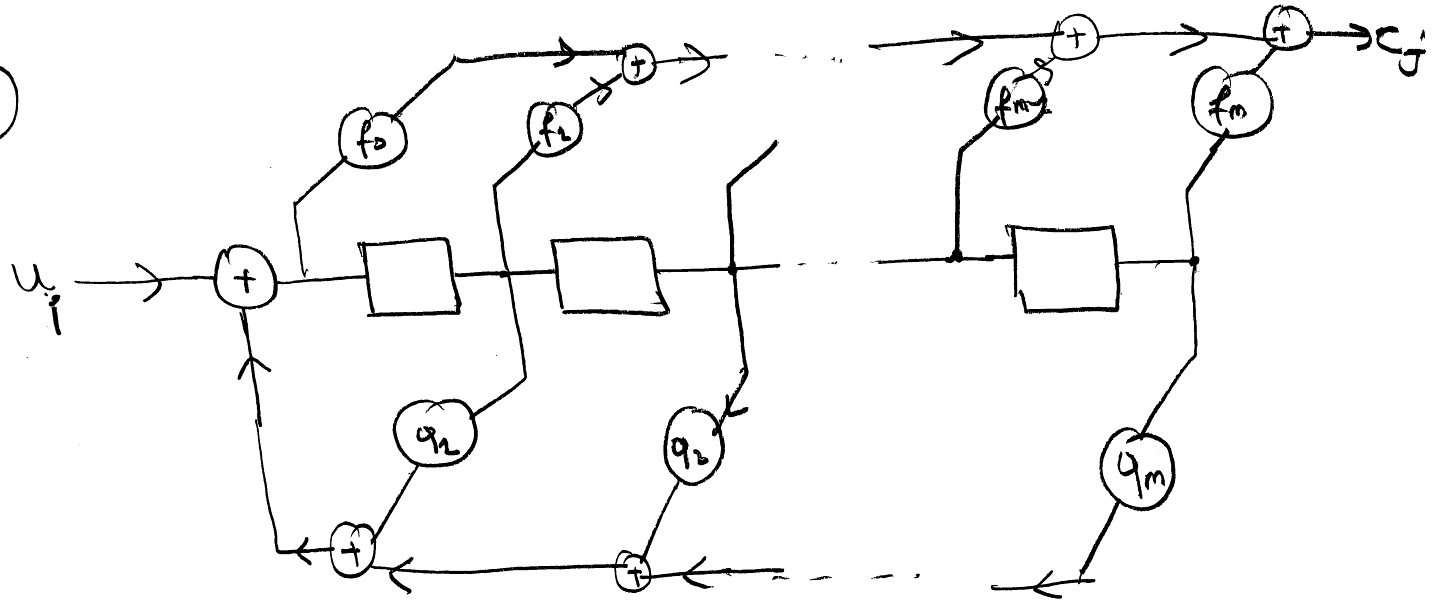
④ Ex:



$G(0) = ?$

Impulse responses ?

(5)

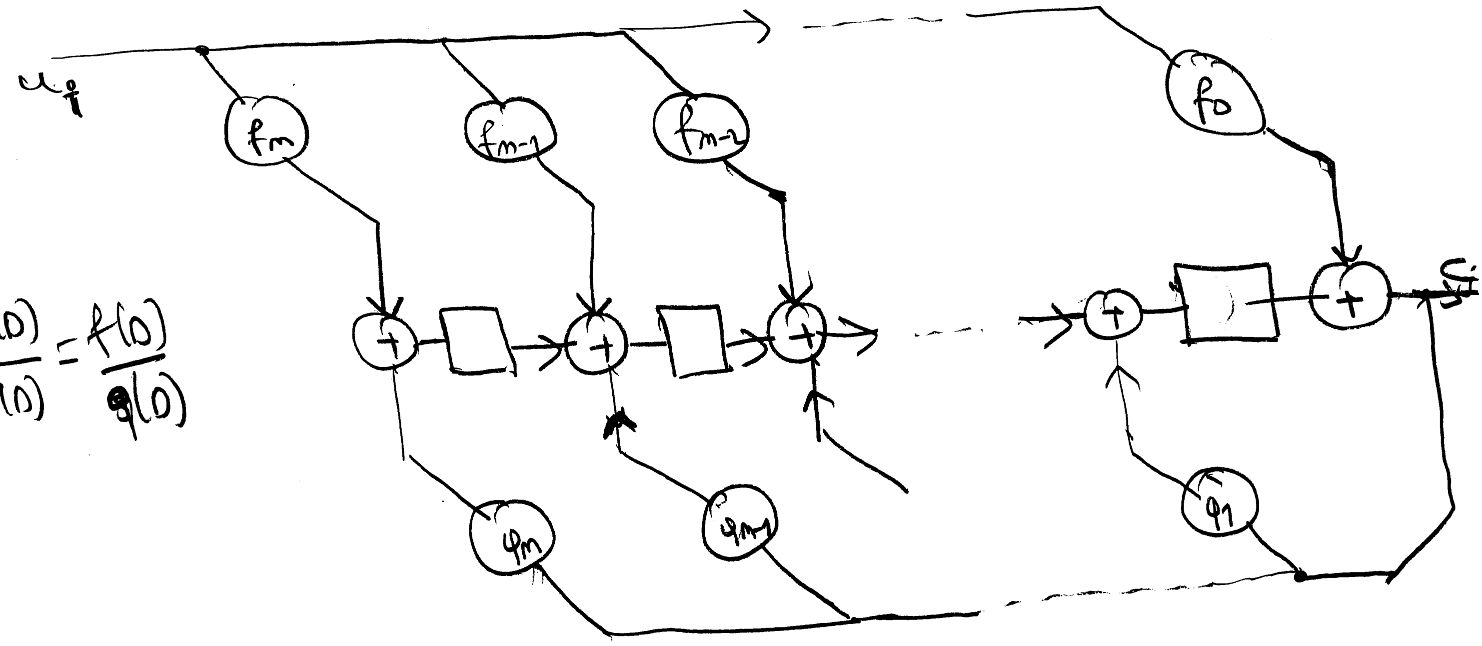


$f(D) = f_0 + f_1 D + \dots f_m D^m$

$q(D) = 1 + q_1 D + \dots + q_m D^m$

$$\frac{c_j(D)}{u_j(D)} = \frac{f(D)}{q(D)}$$

↓ Controller Canonical Form



$$\frac{c_j(D)}{u_j(D)} = \frac{f(D)}{q(D)}$$

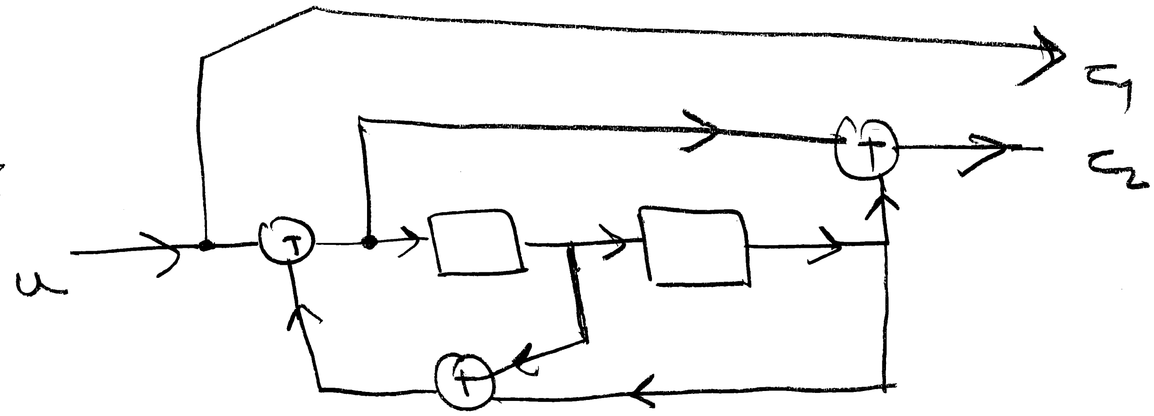
$f(D) = f_0 + f_1 D + \dots f_m D^m$

$q(D) = q_1 D + \dots + q_m D^m$

↓ observer canonical form

5A

Ex:



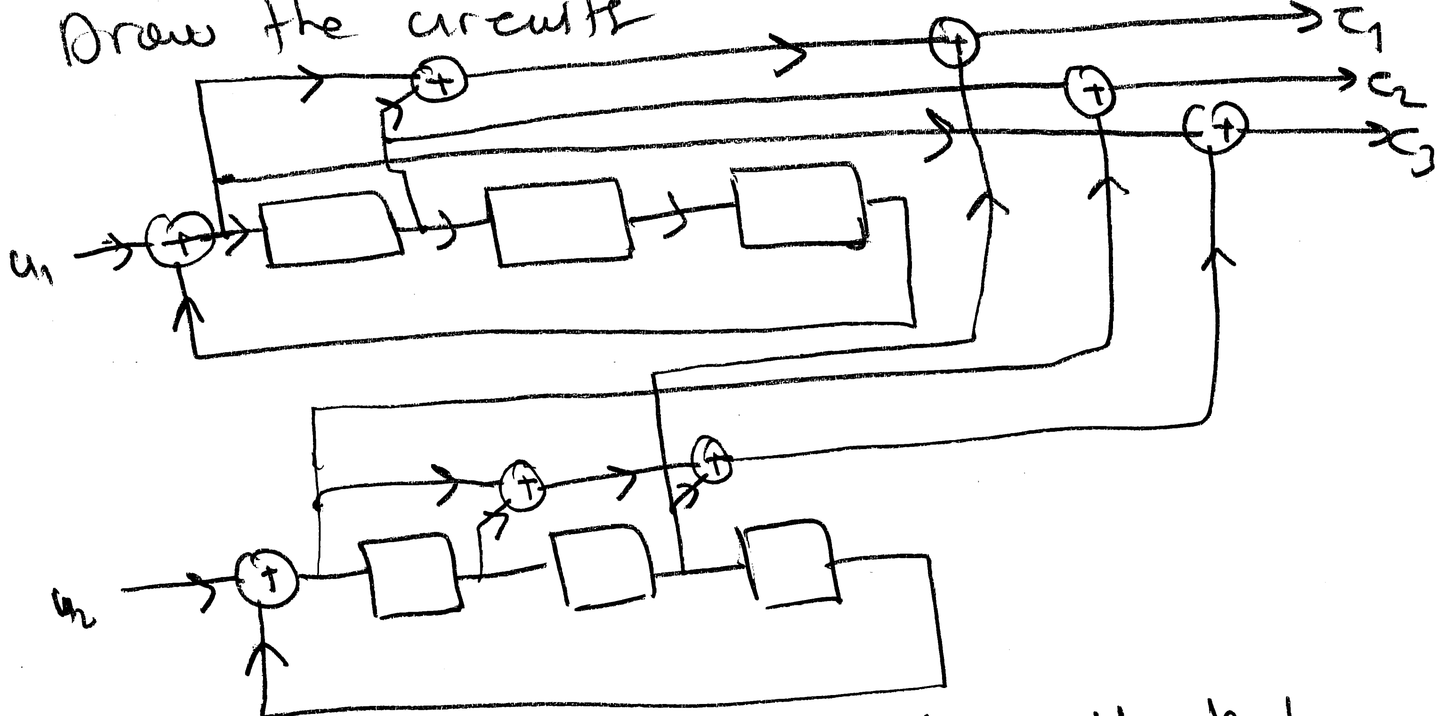
$$G(D) = \left(1 \quad \frac{1+D^2}{1+D+D^2} \right)$$

Ex:

$$G(D) = \begin{pmatrix} \frac{1}{1+D+D^2} & \frac{D}{1+D^3} & \frac{1}{1+D^3} \\ \frac{D^2}{1+D^3} & \frac{1}{1+D^3} & \frac{1}{1+D} \end{pmatrix}$$

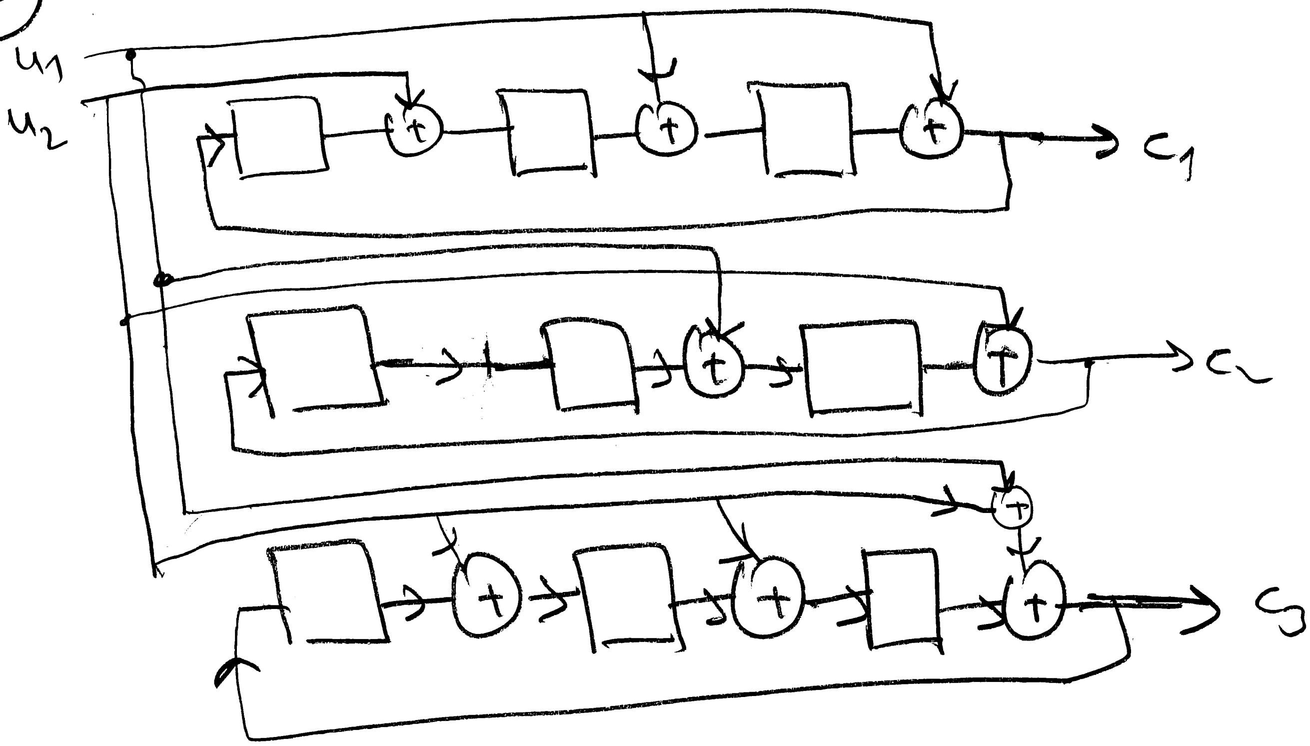
Not controller and observer canonical forms

Draw the circuits



↳ The solution has been taken from the text book "Fundamental of Convolutional Coding, Zigangirov" Page: 37. But the solution seems to be wrong. Draw correct figures.

5B

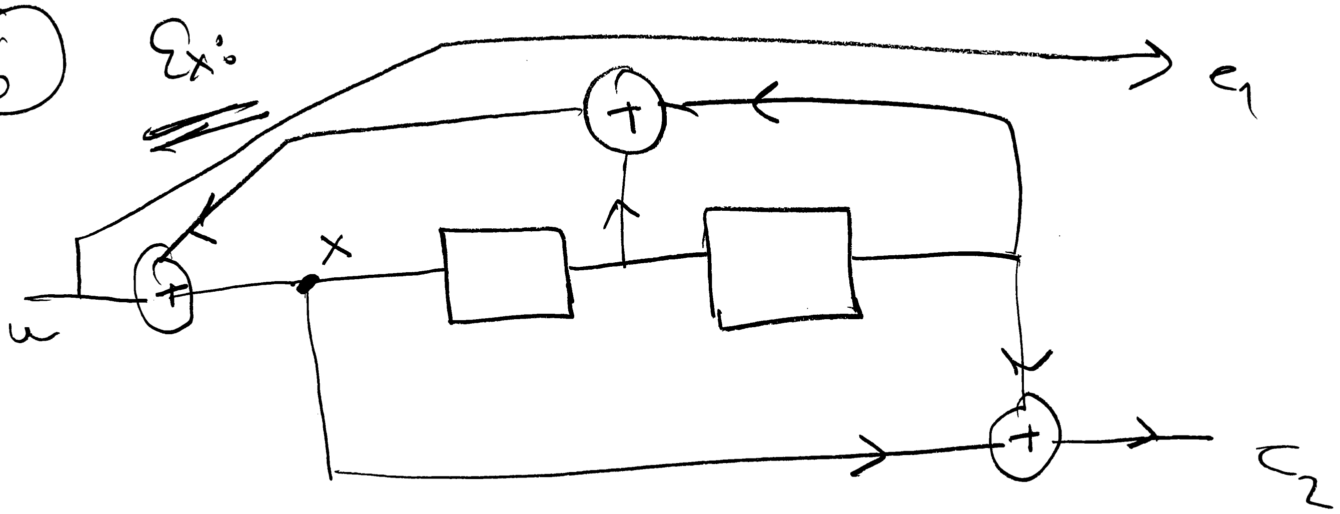


Ex:

$$G(D)=\begin{pmatrix} 1+D & D & 1 \\ D^2 & 1 & 1+D+D^2 \end{pmatrix}$$

Draw encoder circuits.

(6) Ex:



$g_1^1 = 1 \qquad g_{12} = 1\,1\,1\,0\,1\,1\,0\,1\,1\,0\ldots$

$$\left.\begin{array}{l} x_t = u_t + x_{t-1} + x_{t-2} \\ c_{2,t} = x_t + x_{t-2} \end{array}\right\} \longrightarrow u_t = x_t + x_{t-1} + x_{t-2}$$

$c_{1,t} = u_t$

$$c_{2,t} = x_t + x_{t-2}$$

$$c_{2,t-1} = x_{t-1} + x_{t-3}$$

$$c_{2,t-2} = x_{t-2} + x_{t-4}$$

$+$

$$c_{2,t} + c_{2,t-1} + c_{2,t-2} = \underbrace{x_t + x_{t-1} + x_{t-3} + x_{t-4}}_{u_t + u_{t-2}}$$

(7) Apply z-transform

$$\sum c_{2,t} D^t + \sum c_{2,t-1} D^t + \sum c_{2,t-2} D^t$$
$$= \sum u_t D^t + \sum u_{t-2} D^t$$

$$C_2(D) + D C_2(D) + D^2 C_2(D) = u(D) + D^2 u(D)$$

$$C_2(D) = \frac{1+D^2}{1+D+D^2} u(D)$$

$$\frac{C_2(D)}{u(D)} = \frac{1+D^2}{1+D+D^2}$$

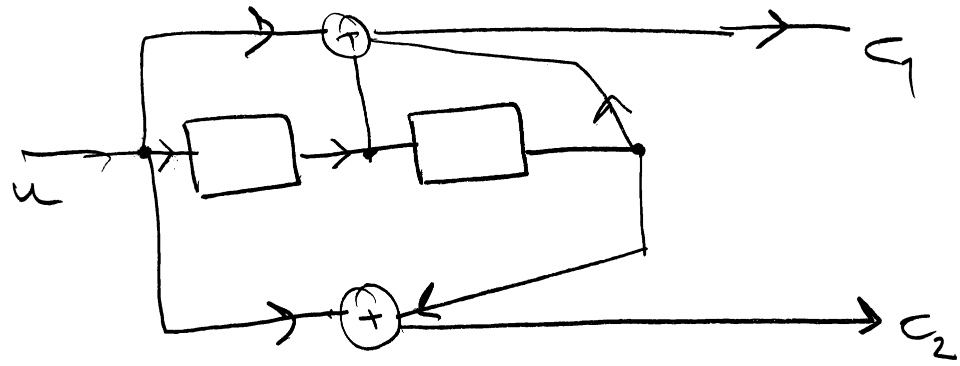
The generator matrix is

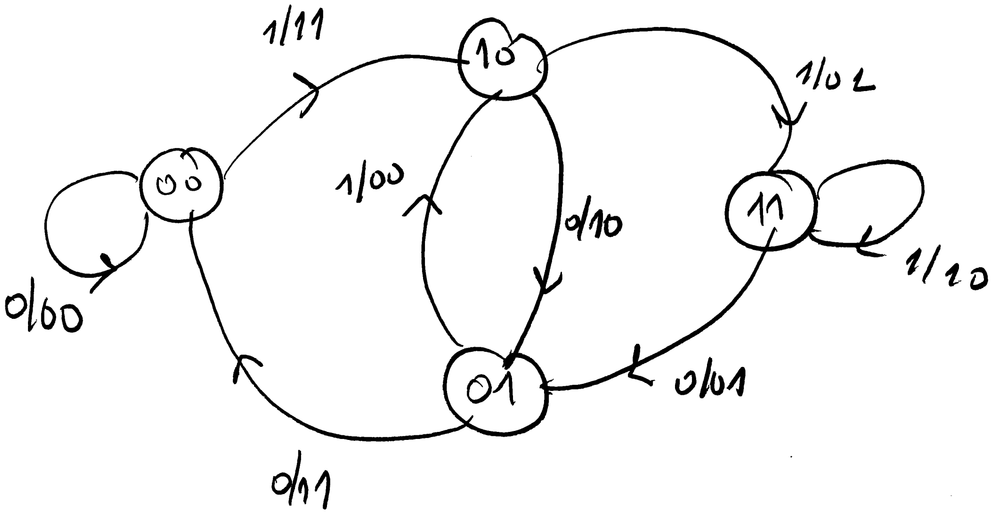
$$G(D) = \left(1 \quad \frac{1+D^2}{1+D+D^2} \right) \rightarrow (1, 5/7)\text{ octal}$$

$$\left(1 \quad \frac{101}{111} \right)$$

8

State Diagram:





Systematic Convolutional Code:

$$G(D) = \begin{pmatrix} 1 & & & G_{1,k+1}(D) & \dots & G_{1,n}(D) \\ & 1 & & & & \\ & & \ddots & & & \\ & & 1 & G_{k,k+1}(D) & \dots & G_{k,n}(D) \end{pmatrix}$$

$$= \left(I_k \mid R(D) \right)$$

$R(D) \to k \times (n-k)$

(8A)

HW

For (1, 5/7) conv- code

(1, 7/5) conv. code

a) Draw encoder circuit

b) Draw state diagram

State Table

c) Encode 10111 using

Trellis diagram, and show

the encoder path in trellis diagram

d) add trellis termination bits to 1011,

so that your encoder terminates

at 0 state

9

Catastrophic Convolutional Codes:

- A catastrophic code maps information sequences with infinite Hamming weight to code sequences with finite Hamming weight.
- For a catastrophic code, a finite number of transmission errors can cause an infinite number of errors in the decoded information sequence. Hence, those codes should be avoided in practice.
- Systematic codes are never catastrophic
- A loop in the state diagram that produces a zero output for a non-zero input is a necessary and sufficient condition for a catastrophic code.

Ex: (111) state has a loop associated with 1/00 in the following encoder