ECE 575 Coding Theory HW # Last

Due: Final Exam Date

(1) $p(x) = x^3 + x + 1$ $p(x) = x^4 + x + 1$ are primitive polynomials.

find the roots of $x^3 + \alpha^8 x^2 + \alpha^{12} x + \alpha = 0$

defined over $GF(2^4)$

(2) Find the determinant of the matrix $A = \begin{bmatrix} 1 & \alpha^4 & \alpha^3 \\ \alpha^2 & 0 & \alpha \\ \alpha^4 & \alpha & \alpha^5 \end{bmatrix}$

over $GF(2^3)$ and $GF(2^4)$

(3) Determine the inverse of the matrix

$$A = \begin{bmatrix} \alpha^{12} & 0 & \alpha \\ 1 & \alpha^8 & \alpha^{14} \\ \alpha^2 & \alpha^{11} & \alpha^5 \end{bmatrix}$$ over $GF(2^3)$ and $GF(2^4)$

(4) Solve the linear equations

$x + \alpha^4 y = \alpha^5$
$\alpha^5 x + \alpha^2 y = \alpha^3$

defined over $GF(2^4)$

(5) Over $GF(2^3)$ a) find generator polynomial of (7,5) single error correcting Reed-Solomon code

b) let $g(x)$ be the generator polynomial
Using $g(x)$ obtain generator matrix $G = [\ ]_{5\times7}$

c) How many bits are used to represent a symbol. Using the result of b) find H

d) $d = (0\ \alpha\ 1\ 0\ \alpha^2) \rightarrow$ dataword
write d as a binary string
encode $d(x)$ using $g(x)$, $c(x)$ is the encoded word
$e = (101\ 000\ 000\ 000\ 000\ 000\ 000) \rightarrow$ write $e(x)$
$r(x) = c(x) + e(x)$ decode $r(x)$

(6) Repeat problem 5 for a decoder that uses the Berlekamp algorithm for determining the error locations and the error-evaluator algorithm to determine the error locations.

(7) For a Reed Solomon code we have the equation set

$$S_1 = Y_1 X_1 + Y_2 X_2 + \cdots Y_t X_t$$
$$S_2 = Y_1 X_1^2 + Y_2 X_2^2 + \cdots Y_t X_t^2$$
$$\vdots$$
$$S_{2t} = Y_1 X_1^{2t} + Y_2 X_2^{2t} + \cdots Y_t X_t^{2t}$$

which can be written in short as

$$S_i = \sum_{J=1}^{t} Y_J X_J^{2i} \qquad i = 1 \ldots 2t$$

$X_1, \ldots X_t$ are the error location numbers.

$$\sigma(x) = 1 + \sigma_1 x + \sigma_2 x^2 + \cdots \sigma_\nu x^\nu = \prod_{K=1}^{t} (1 - x X_k)$$

Starting from $\sigma(x)$ show that the error location numbers can be found from

$$\begin{bmatrix} S_1 & S_2 & \cdots & S_t \\ S_2 & S_3 & \cdots & S_{t+1} \\ \vdots & & & \\ S_t & S_{t+1} & \cdots & S_{2t-1} \end{bmatrix} \begin{bmatrix} \sigma_t \\ \sigma_{t-1} \\ \vdots \\ \sigma_1 \end{bmatrix} = \begin{bmatrix} -S_{t+1} \\ -S_{t+2} \\ \vdots \\ -S_{2t} \end{bmatrix}$$

Hint:
Google wikipedia Reed Solomon code