ECE 575 HW 1

(1) $S=\{0,1,x,x+1\}$ is a polynomial set

$p(x)\in S,\ q(x)\in S \quad p(x)\oplus q(x)\rightarrow \oplus$: coefficients of the polynomials are summed using mod 2 summation

i.e, $x+x=2x$
$=0$

a) Show that $(S,\oplus)$ form a Group

b) Find the inverse of the elements x & $x+1$

(2) $S=\{0,1,2,3\}$ $\oplus \rightarrow$ mod 4 addition
$\otimes \rightarrow$ mod 4 multiplication

is $(S,\oplus,\otimes)$ a field or not?

(3) $S=\{0,1,x,x+1,x^2,x^2+1,x^2+x,x^2+x+1\}$

Coefficients of the polynomials in S are selected from GF(2) (Galois field 2)

i.e. while summing two polynomials or multiplying two polynomials we use mod 2 operation for the resulting polynomial coefficients.

i.e, $(x+1)(x+1)=x^2+2x+1$
$2\equiv 0$
$=x^2+1$

Now we define two operations $\oplus$, $\otimes$

$p(x), q(x)\in S$

$p(x)\oplus q(x)\rightarrow$ remainder of $p(x)+q(x)$ after division by x^3+1+x

$p(x)\otimes q(x)\rightarrow$ remainder of $p(x).q(x)$ after division by x^3+1+x

Show that $(S,\oplus,\otimes)$ is a field.